

# THE SUMMATION-BY-PARTS FRAMEWORK AND CFD'S HOLY GRAIL: NON-LINEAR STABILITY AND ROBUSTNESS TRACK NUMBER 700

MATTEO PARSANI\*, DAVID C. DEL REY FERNÁNDEZ†, JASON HICKEN\*\* AND  
ANDREW R. WINTERS††

\* King Abdullah University of Science & Technology (KAUST)  
Computer Electrical and Mathematical Science and Engineering Division (CEMSE)  
Extreme Computing Research Center (ECRC)  
23955-6900, Thuwal, Saudi Arabia  
matteo.parsani@kaust.edu.sa, <https://ecrc.kaust.edu.sa/Pages/Parsani.aspx>

† National Institute of Aerospace (NIA) & NASA Langley Research Center  
Hampton, Virginia, United States  
dcdelrey@gmail.com

\*\* Rensselaer Polytechnic Institute (RPI)  
110 Eighth Street Troy, NY USA 12180  
hickej2@rpi.edu, <https://faculty.rpi.edu/node/36040>

†† Linköping University (LiU)  
Computational Mathematics  
581 83 Linköping, Sweden  
andrewthewinters@gmail.com

**Key words:** Computational fluid dynamics, Summation-by-parts, finite-differences, discontinuous/continuous Galerkin, flux reconstruction, high-order methods, entropy stability,  $p$ -,  $h$ - and  $hp$ -adaptivity.

## ABSTRACT

The summation-by-parts (SBP) concept was originally developed in the finite difference community with the goal of mimicking finite element energy analysis techniques. In recent years, this simple idea has been exponentially generalized enabling a unifying framework for the stability analysis of many spatial discretizations including finite difference, finite volume, flux reconstruction, and continuous/discontinuous Galerkin (DG) methods on structured and unstructured polytope meshes for linear and non-linear conservation laws on conforming and non-conforming grids. The most important consequence of SBP is that it naturally guides the path to stability and robustness as it mimics continuous stability analysis. The SBP concept provides a strong theoretical framework, that is discretization agnostic, for the analysis of existing schemes and the design of flexible high-order numerical approximations that are robust for complex multi-scale applications, e.g. compressible turbulence. This robustness is guaranteed via discrete entropy stability. However, the SBP framework can incorporate other desirable properties that are potentially important for CFD and beyond: For instance, it is possible to construct split-form SBP methods,

---

for the compressible Euler equations, that are fully conservative and kinetic energy preserving. These new developments offer exciting novel ways of incorporating sophisticated turbulence modelling in e.g. the DG framework. Moreover, these robustness investigations guide the way to the construction of provably stable complex boundary conditions for fluid dynamics which remain a major obstacle on the path towards complete entropy stability. This mini symposium will bring together experts from different disciplines that push the boundaries of SBP related methods to new frontiers with talks on adaptivity, HPC, non-linear robustness, high order turbulence modeling, boundary conditions, efficient time integration for high-order methods, meshing technologies, and general new developments of the SBP concept itself.

We invite contributions addressing fundamental and or applied aspects of SBP methods. We anticipate contributions in the following key areas:

- Entropy stable algorithms
- Linearly stable algorithms
- $h/p$ -adaptation
- Methods for moving mesh problems
- Development of SBP differentiation matrices
- SBP in time
- Efficient SBP methods for HPC
- Boundary conditions