Vibration and Wave Dispersion Analysis of Finite Granular Beams - Discrete versus Continuum approaches

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Abstract:

The present study theoretically investigates the free vibration problem of a discrete granular system and the wave dispersion analysis of such a system. This microstructured system consists of uniform grains elastically connected by shear and rotational springs. In the presence of length scales, the elastic wave propagation problem involves an interplay between wave dispersion and structural features. This discrete repetitive system could be considered as a discrete Cosserat chain or a lattice elastic model with shear interaction. It is shown that for infinite number of grains, the deflection equation of the granular system leads to the fourth-order differential equation of Bresse-Timoshenko obtained for continuous beam with shear deformation. The eigenfrequencies of this discrete system are exactly calculated for the granular beam resting on two simply supports, starting from the resolution of the linear difference eigenvalue problem. Next, the discrete model is continualized by approximate methods of Taylor polynomial expansion and Padé rational expansion. Scale effects of the granular chain are clearly captured by the continuous gradient elasticity model. The natural frequencies of the continuous gradient Cosserat models are compared with those of the discrete Cosserat model associated with the granular chain. Scale effects and wave dispersion characteristics of the granular chain are clearly captured by the continuous gradient elasticity model.

Keywords: Vibration analysis; Cosserat continuum; Wave dispersion; Lattice formulation; Timoshenko beam; Difference equations; Enriched continua